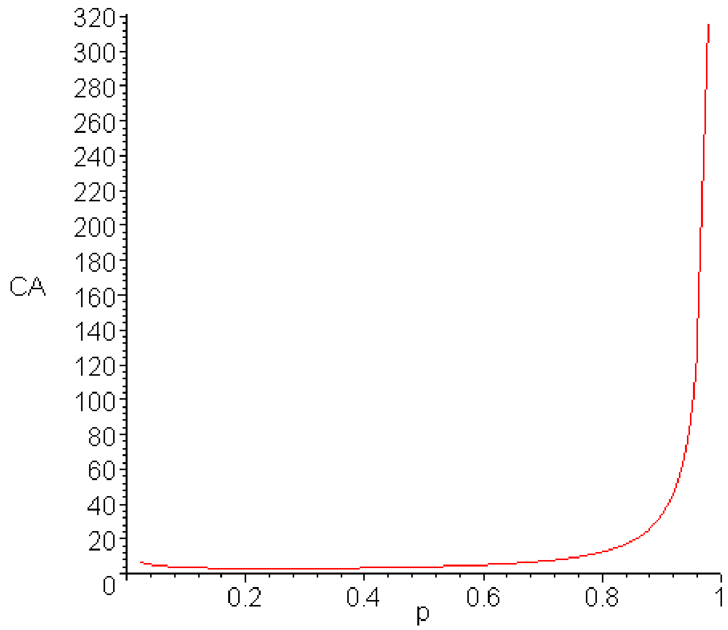
Exercice #1

$X \sim Ber(p)$
Coefficient d'asymétrie (CA) $= \frac{E(X^3)}{\sigma^3}$
$E(X) = p$
$E(X^2) = 0^2(1-p) + 1^2 p = p$
$Var(X) = E(X^2) - E(X) = p - p^2 \quad \Rightarrow \quad \sigma^3 = (p(1-p))^{\frac{3}{2}}$
$E(X^3) = 0^3(1-p) + 1^3 p = p \quad \Rightarrow \quad CA = \frac{1}{\sqrt{p(1-p)^3}}$



Exercice #2

$L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$
$l(\lambda) = \sum_{i=1}^{n} ln\left(\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right) = -\lambda n + \sum_{i=1}^{n} ln\lambda - \sum_{i=1}^{n} x_i!$
$\frac{dl(\lambda)}{d\lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i = 0 \quad \Rightarrow \quad \frac{1}{\lambda}\sum_{i=1}^{n} x_i = n \quad \Rightarrow \quad \widehat{\lambda}_{MV} = \overline{x}$

Exercice #3

a) $H_0 : \mu = \mu_0 = 0.323$
$H_1 : \mu \neq \mu_0$
$z_{\alpha/2} = 1.96$
$z_0 = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{0.314-0.323}{0.02/5} = -2.25$
$|z_0| > z_{\alpha/2} \quad \Rightarrow$ Au seuil de 5%, $\mu \neq 0.323$

b) $\beta = P(\text{Accepter } H_0 \mid H_0 \text{ est fausse})$
Zone d'acceptation de $H_0 \mid H_0$ vraie : 0.323 ± 0.0078
$\beta = P(\overline{X} \in [0.3152, 0.3308] \mid \mu = 0.313)$
$\overline{X} \sim N(0.313, (0.02/5)^2)$
$Z \sim N(0,1)$
$\beta = P\left(\frac{0.3152-0.313}{0.02/5} \leq Z \leq \frac{0.3308-0.313}{0.02/5}\right) = \Phi(4.45) - \Phi(0.55) = 0.29116$
$Puissance = 1 - \beta = 0.70884$

Exercice #4

a) $\widehat{p}_i = \frac{x_i}{n}$

Intervalles simultanés : $\frac{\alpha/2}{5} = 0.025/5 = 0.005 = \alpha_i/2$

$p_i \in \widehat{p}_i \pm z_{\alpha_i/2}\sqrt{\frac{\widehat{p}_i(1-\widehat{p}_i)}{n}}$ avec une probabilité de 0.995

$z_{0.005} = 2.5758 \qquad n = 1000$

Intervalles de confiance simultanés au niveau de 95% pour...

$p_0 : 0.047 \pm 0.017$

$p_1 : 0.051 \pm 0.018$

$p_2 : 0.059 \pm 0.019$

$p_3 : 0.107 \pm 0.025$

$p_4 : 0.736 \pm 0.036$

b) Intervalles simultanés : $\frac{\alpha/2}{4} = 0.025/4 = 0.00625 = \alpha_i/2$

$p_i - p_j \in \widehat{p}_i - \widehat{p}_j \pm z_{\alpha_i/2}\sqrt{\frac{\widehat{p}_i(1-\widehat{p}_i)}{n_i} + \frac{\widehat{p}_j(1-\widehat{p}_j)}{n_j}}$ avec une probabilité de 0.99375

$z_{0.00625} = 2.4977 \qquad n_i = n_j = 1000$

Intervalles de confiance simultanés au niveau de 95% pour...

$p_4 - p_0 : 0.689 \pm 0.039$

$p_4 - p_1 : 0.685 \pm 0.039$

$p_4 - p_2 : 0.677 \pm 0.039$

$p_4 - p_3 : 0.629 \pm 0.043$

c) Estimation de θ :

$P_{H_0}(X = 0) = (1-\theta)^4$

$P_{H_0}(X = 1) = 4\theta(1-\theta)^3$

$P_{H_0}(X = 2) = 6\theta^2(1-\theta)^2$

$P_{H_0}(X = 3) = 4\theta^3(1-\theta)$

$P_{H_0}(X = 4) = \theta^4$

$L(\theta) = \frac{1000!}{\prod_{i=0}^{4} x_i!}\prod_{i=0}^{4} p_i^{x_i} = \frac{1000!(1-\theta)^{4\cdot 47}(4\theta(1-\theta)^3)^{51}(6\theta^2(1-\theta)^2)^{59}(4\theta^3(1-\theta))^{107}\theta^{4\cdot 736}}{47!\ 51!\ 59!\ 107!\ 736!}$

$l'(\theta) = \frac{d}{d\theta}(566\ln(1-\theta) + 3434\ln\theta) = \frac{-566}{1-\theta} + \frac{3434}{\theta} = 0$

$\Rightarrow \widehat{\theta}_{MV} = \frac{1717}{2000} = 0.8585$

		0	1	2	3	4
O_i	fréquence	47	51	59	107	736
T_i	bin$(4, \widehat{\theta})$	0.4	9.73	88.54	358.1	543.2

$\sum_{i=0}^{4} \frac{(O_i - T_i)^2}{T_i} \sim \chi_v^2$

v =(nombre de classes) - 1 - (nombre de paramètres à estimer) = 3

$\chi^2_{3,0.05} = 7.81$

$\sum_{i=0}^{4} \frac{(O_i - T_i)^2}{T_i} = \frac{(47-0.4)^2}{0.4} + \frac{(51-9.73)^2}{9.73} + \frac{(59-88.54)^2}{88.54} + \frac{(107-358.1)^2}{358.1} + \frac{(736-543.2)^2}{543.2}$

$\frac{(47-0.4)^2}{0.4} = 5428.9 \quad \Rightarrow \quad \sum_{i=0}^{4} \frac{(O_i - T_i)^2}{T_i} >> 7.81$

$\Rightarrow$ On rejette H_0 .