Numéro 1

a) $\underline{N = m + n \quad \text{impair}}$

n tirages sans remise

U_j : numéro d'une boule tirée au j^e tirage.

$U_j \in \{1, 1, 2, 2, \cdots, \frac{N-1}{2}, \frac{N-1}{2}, \frac{N+1}{2}\}$

$H_0 : T = \sum_{i=1}^{n} U_i$

$\mathrm{E}(U_j) = \sum_{i=1}^{N} x_i \mathrm{P}(U_j = x_i) = \frac{1}{N}\left(2 \times \frac{(\frac{N-1}{2})(\frac{N-1}{2}+1)}{2} + \frac{N+1}{2}\right)$

$\mathrm{E}(U_j) = \frac{N}{4} + \frac{1}{2} + \frac{1}{4N} = \frac{N^2+2N+1}{4N}$

$\mathrm{E}(T) = \sum_{i=1}^{n} \mathrm{E}(U_i) = \frac{n(N^2+2N+1)}{4N}$

$\underline{N \text{ pair}}$

$U_j \in \{1, 1, 2, 2, \cdots, \frac{N}{2}, \frac{N}{2}\}$

$\mathrm{E}(U_j) = \frac{1}{N}\left(2 \times \frac{\frac{N}{2}(\frac{N}{2}+1)}{2}\right) = \frac{1}{N}\left(\frac{N^2}{4} + \frac{N}{2}\right) = \frac{N+2}{4}$

$\Rightarrow \quad \mathrm{E}(T) = \frac{n(N+2)}{4}$

b) $\underline{N \text{ impair}}$

$\mathrm{Var}\,(T) = \mathrm{Var}\left(\sum_{i=1}^{n} U_i\right) = \sum_{k=1}^{n} \mathrm{Var}\,(U_k) + \sum_{1 \le i \ne j \le n} \mathrm{Cov}(U_i, U_j)$

$\mathrm{Var}\,(U_k) = \mathrm{E}(U_k^2) - (\mathrm{E}(U_k))^2$

$\mathrm{E}(U_k^2) = \sum_{i=1}^{N} x_i^2 \mathrm{P}(U_k = x_i) = \frac{1}{N}\left(\frac{2(\frac{N-1}{2})(\frac{N-1}{2}+1)(2 \times \frac{N-1}{2}+1)}{6} + \left(\frac{N+1}{2}\right)^2\right)$

$\mathrm{E}(U_k^2) = \frac{(N+1)(N^2+2N+3)}{12N}$

$\Rightarrow \quad \mathrm{Var}\,(U_k) = \frac{(N-1)(N+1)(N^2+3)}{48N^2}$

$\mathrm{Cov}(U_i, U_j) = \mathrm{E}(U_i U_j) - \mathrm{E}(U_i)\mathrm{E}(U_j)$

$\mathrm{E}(U_i U_j) = \sum_{k=1}^{\frac{N+1}{2}} \mathrm{E}(U_i U_j | U_i = k)\mathrm{P}(U_i = k) = \sum_{k=1}^{\frac{N+1}{2}} k\mathrm{E}(U_j | U_i = k)\mathrm{P}(U_i = k)$

$\mathrm{E}(U_i U_j) = \sum_{k=1}^{\frac{N-1}{2}} k\mathrm{E}(U_j | U_i = k)\mathrm{P}(U_i = k) + \frac{N+1}{2}\mathrm{E}(U_j | U_i = \frac{N+1}{2})\mathrm{P}(U_i = \frac{N+1}{2})$

$\mathrm{E}(U_i U_j) = \sum_{k=1}^{\frac{N-1}{2}} k \times \frac{2(1 + \cdots + \frac{N-1}{2}) - k + \frac{N+1}{2}}{N-1} \times \frac{2}{N} + \frac{N+1}{2} \times \frac{2(1 + \cdots + \frac{N-1}{2})}{N-1} \times \frac{1}{N}$

En développant correctement (par Maple) :

```
factor(sum(k*(2*sum(j,j=1..(N-1)/2)-k+(N+1)/2)/(N-1)*2/N,
k=1..(N-1)/2)+(N+1)*sum(i,i=1..(N-1)/2)/(N-1)/N);
```

$\mathrm{E}(U_i U_j) = \frac{(N+1)(3N^2+8N+9)}{48N}$

$$\Rightarrow \quad \mathrm{Cov}(U_i, U_j) = \frac{-(N+1)(N^2+3)}{48N^2}$$
$$\mathrm{Var}\,(T) = n\mathrm{Var}\,(U_k) + n(n-1)\mathrm{Cov}(U_i, U_j)$$
$$\mathrm{Var}\,(T) = \frac{n(N+1)(N^2+3)(N-n)}{48N^2}$$

<u>N pair</u>

$$\mathrm{Var}\,(T) = \mathrm{Var}\left(\sum_{i=1}^{n} U_i\right) = \sum_{k=1}^{n} \mathrm{Var}\,(U_k) + \sum_{1\le i\ne j\le n} \mathrm{Cov}(U_i, U_j)$$
$$\mathrm{Var}\,(U_k) = \mathrm{E}(U_k^2) - (\mathrm{E}(U_k))^2$$
$$\mathrm{E}(U_k^2) = \sum_{i=1}^{N} x_i^2 \mathrm{P}(U_k = x_i) = \frac{2}{N}\left(\frac{\frac{N}{2}(\frac{N}{2}+1)(N+1)}{6}\right) = \frac{(N+1)(N+2)}{12}$$
$$\Rightarrow \quad \mathrm{Var}\,(U_k) = \frac{(N-2)(N+2)}{48}$$
$$\mathrm{Cov}(U_i, U_j) = \mathrm{E}(U_iU_j) - \mathrm{E}(U_i)\mathrm{E}(U_j)$$
$$\mathrm{E}(U_iU_j) = \sum_{k=1}^{\frac{N}{2}} \mathrm{E}(U_iU_j|U_i = k)\mathrm{P}(U_i = k) = \sum_{k=1}^{\frac{N}{2}} k\mathrm{E}(U_j|U_i = k)\mathrm{P}(U_i = k)$$
$$\mathrm{E}(U_iU_j) = \sum_{k=1}^{\frac{N}{2}} k \times \frac{2(1+\cdots+\frac{N}{2}) - k}{N-1} \times \frac{2}{N}$$

En développant encore correctement :

```
factor(2/N*sum(k*(2*sum(i,i=1..(N/2))-k)/(N-1),k=1..(N/2)));
```

$$\mathrm{E}(U_iU_j) = \frac{(N+2)(3N^2+2N-4)}{48(N-1)}$$
$$\Rightarrow \quad \mathrm{Cov}(U_i, U_j) = \frac{-(N-2)(N+2)}{48(N-1)}$$
$$\mathrm{Var}\,(T) = n\mathrm{Var}\,(U_k) + n(n-1)\mathrm{Cov}(U_i, U_j)$$
$$\mathrm{Var}\,(T) = \frac{n(N-2)(N+2)(N-n)}{48(N-1)}$$

c) $N = 18 \qquad n = 10$

$$T = 1+2+3+5+7+9+6+2+1 = 40$$
$$\mathrm{E}(T) = \frac{n(N+2)}{4} = \frac{10\times 20}{4} = 50$$
$$\mathrm{Var}\,(T) = \frac{n(N-2)(N+2)(N-n)}{48(N-1)} = \frac{10\times 16\times 20\times 8}{48\times 17} = 31.37$$
$$\mathrm{P}(T \le 40) \approx \mathrm{P}(T \le 40.5) = \mathrm{P}\left(\frac{T-50}{\sqrt{31.37}} \le -1.696\right) \simeq \Phi(-1.696) = 0.04492797 < 0.05$$

$\Rightarrow$ Au seuil de 5%, on rejette H_0. Donc la distribution G est vraiment plus aplatie.

d) On rejette H_0 si $F_0 = \dfrac{s_B^2}{s_A^2} > F_{\alpha, n_B-1, n_A-1}$

e) $n_A = 8 \qquad n_B = 10$

$$F_0 \sim F_{n_B-1, n_A-1}$$
$$\mathrm{E}(F_0) = \frac{2}{n_A - 1 - 2} = 0.4$$
$$\sqrt{\mathrm{Var}\,(F_0)} = \sqrt{\frac{2(n_A-1)(n_A+n_B-4)}{(n_B-1)(n_A-3)^2(n_A-5)}} = \sqrt{\frac{2\times 7\times 14}{9\times 25\times 3}} = 0.53886$$

$F_0 = 3.098743$
$\mathrm{P}(F_0 \geq 3.098743) = 0.07501103 > 0.05$
$\Rightarrow$ Au seuil de 5%, on ne rejette pas H_0. Donc la distribution G n'est pas plus aplatie.

Numéro 2

a) $\overline{\theta} = \overline{Z} = \dfrac{1}{10}\sum_{i=1}^{10} Z_i = 26.8443$

$\theta \in \overline{\theta} \pm t_{\alpha/2,N-1}\sqrt{\dfrac{s^2}{N}} = 26.8443 \pm t_{0.05,9}\sqrt{\dfrac{3.985836}{10}}$
$\theta \in 26.8443 \pm 1.833113 \times 0.6313 = [25.687, 28.002]$
Valide si les Z_i sont iid avec moyenne θ et variance σ^2 finie, et si N est grand.

b) $\widehat{\theta} = \underset{1\leq i\leq j\leq N}{\text{méd}} \left(\dfrac{Z_i + Z_j}{2}\right) = 26.746$

Programme S-Plus utilisé :

```
x<-c(25.826,27.427,30.503,26.854,25.043,26.638,23.418,27.022,
26.441,29.271)
y<-matrix(rep(0,100),ncol=10)
i<-1
while (i<=10) {
    j<-1
    while (j<=i) {
        y[i,j]<-x[i]/2 + x[j]/2
        j<-j+1
        }
    i<-i+1
    }
y<-y[y!=0]
median(y)

y<-sort(y)
y[12]
y[44]
```

$\theta \in [A_{(k+1)}, A_{(M-k)}] \qquad M = 55$
$p_i = \mathrm{P}_{H_0}(V_s = i)$

$$\sum_{i=0}^{k} p_i \approx \alpha/2 = 0.05$$

$$\mathrm{P}(V_s \leq 11) = 0.0527 = \sum_{i=0}^{11} p_i$$

$\Rightarrow \quad \theta \in [A_{(12)}, A_{(44)}] = [25.826, 28.0625]$

y à la fin de la boucle, c'est-à-dire les $\frac{Z_i+Z_j}{2}$: (page suivante)

```
> y
          [,1]    [,2]    [,3]    [,4]    [,5]
 [1,] 25.8260  0.0000  0.0000  0.0000  0.0000
 [2,] 26.6265 27.4270  0.0000  0.0000  0.0000
 [3,] 28.1645 28.9650 30.5030  0.0000  0.0000
 [4,] 26.3400 27.1405 28.6785 26.8540  0.0000
 [5,] 25.4345 26.2350 27.7730 25.9485 25.0430
 [6,] 26.2320 27.0325 28.5705 26.7460 25.8405
 [7,] 24.6220 25.4225 26.9605 25.1360 24.2305
 [8,] 26.4240 27.2245 28.7625 26.9380 26.0325
 [9,] 26.1335 26.9340 28.4720 26.6475 25.7420
[10,] 27.5485 28.3490 29.8870 28.0625 27.1570
          [,6]    [,7]    [,8]   [,9]  [,10]
 [1,]  0.0000  0.0000  0.0000  0.000  0.000
 [2,]  0.0000  0.0000  0.0000  0.000  0.000
 [3,]  0.0000  0.0000  0.0000  0.000  0.000
 [4,]  0.0000  0.0000  0.0000  0.000  0.000
 [5,]  0.0000  0.0000  0.0000  0.000  0.000
 [6,] 26.6380  0.0000  0.0000  0.000  0.000
 [7,] 25.0280 23.4180  0.0000  0.000  0.000
 [8,] 26.8300 25.2200 27.0220  0.000  0.000
 [9,] 26.5395 24.9295 26.7315 26.441  0.000
[10,] 27.9545 26.3445 28.1465 27.856 29.271
```

c) $\tilde{\theta} = \underset{1 \le i \le N}{\text{méd}} (Z_i) = 26.746$

$\theta \in [Z_{(K+1)}, Z_{(N-k)}]$

$\sum_{j=0}^{k} \binom{N}{j} \frac{1}{2^N} \approx \alpha/2 = 0.05$

$\sum_{j=0}^{2} \binom{10}{j} \frac{1}{2^{10}} = 0.05469$

$\Rightarrow \quad \theta \in [Z_{(3)}, Z_{(8)}] = [25.826, 27.427]$

Valide si les Z_i sont iid avec loi continue avec médiane unique θ.

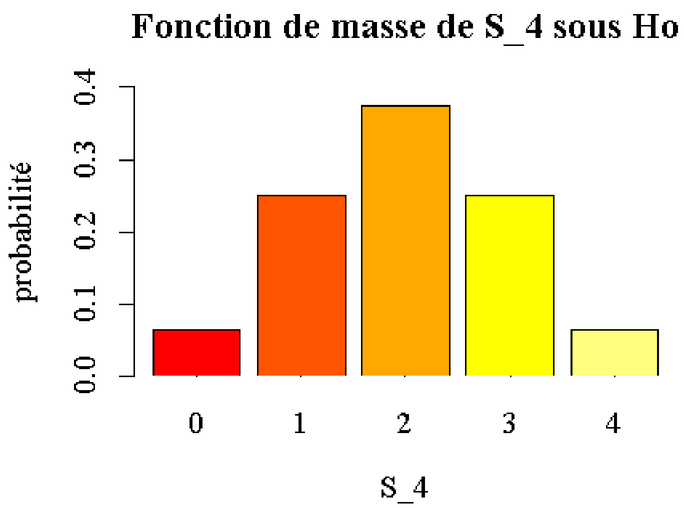
Problèmes du livre

1(i) $\mathrm{P}_{H_0}(S_4 = a) = \binom{4}{a} \frac{1}{2^4} = \mathrm{p}(a)$

$\mathrm{p}(0) = \mathrm{p}(4) = \frac{1}{16}$

$\mathrm{p}(1) = \mathrm{p}(3) = \frac{1}{4}$

$\mathrm{p}(2) = \frac{3}{8}$

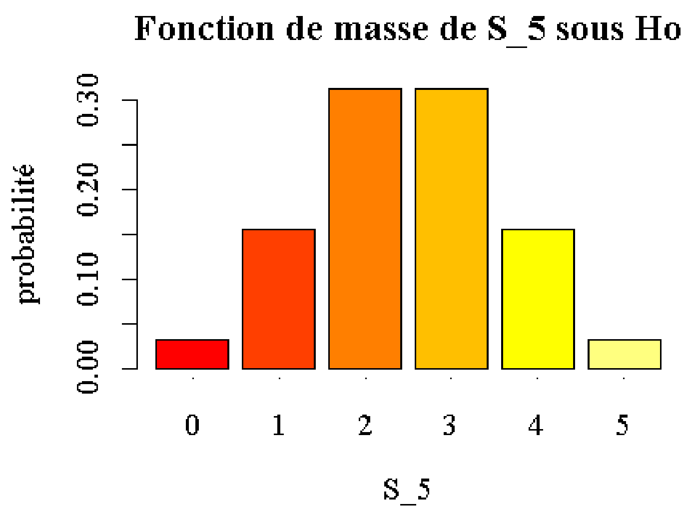


1(ii) $P_{H_0}(S_5 = a) = \binom{5}{a}\frac{1}{2^5} = p(a)$

$p(0) = p(5) = \frac{1}{32}$

$p(1) = p(4) = \frac{5}{32}$

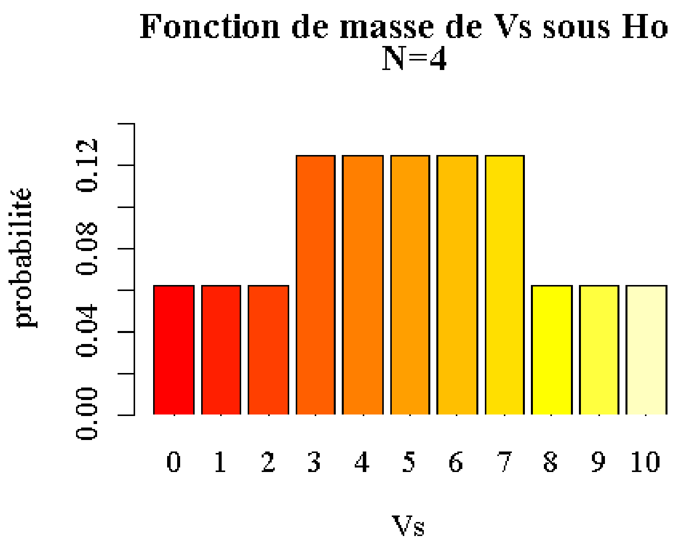
$p(2) = p(3) = \frac{5}{16}$



17(i)

Rangs signés v	-1,-2,-3,-4 0	+1,-2,-3,-4 1	-1,+2,-3,-4 2	-1,-2,+3,-4 3
Rangs signés v	-1,-2,-3,+4 4	+1,+2,-3,-4 3	+1,-2,+3,-4 4	+1,-2,-3,+4 5
Rangs signés v	-1,+2,+3,-4 5	-1,+2,-3,+4 6	-1,-2,+3,+4 7	+1,+2,+3,-4 6
Rangs signés v	+1,+2,-3,+4 7	+1,-2,+3,+4 8	-1,+2,+3,+4 9	+1,+2,+3,+4 10

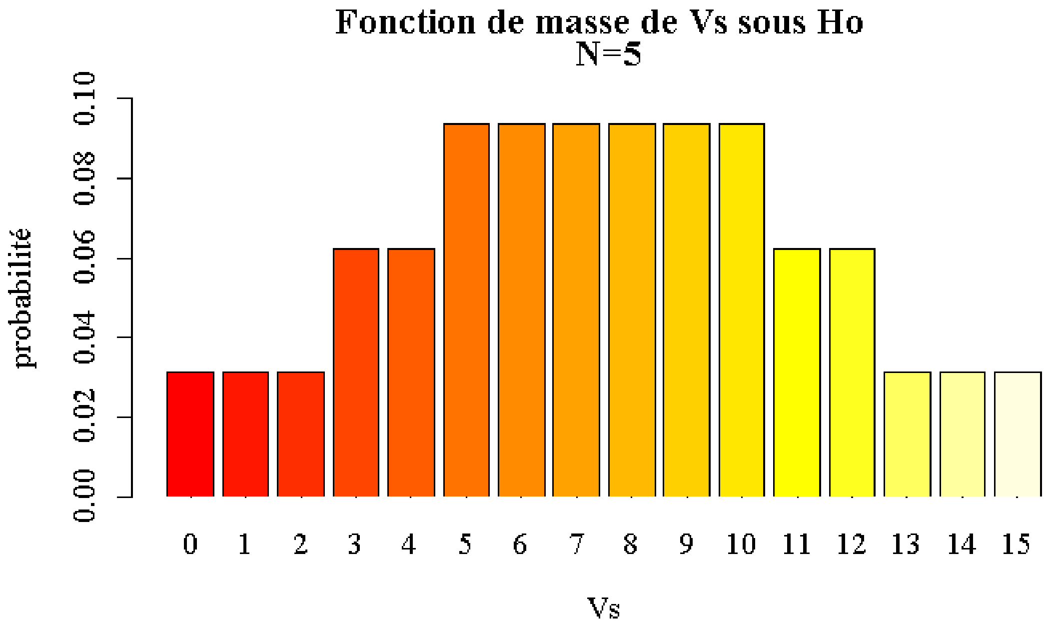
v	0	1	2	3	4	5	6	7	8	9	10
$P_{H_0}(V_s = v)$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$



17(ii)

Rangs signés v	-1,-2,-3,-4,-5 0	+1,-2,-3,-4,-5 1	-1,+2,-3,-4,-5 2	-1,-2,+3,-4,-5 3
Rangs signés v	-1,-2,-3,+4,-5 4	-1,-2,-3,-4,+5 5	+1,+2,-3,-4,-5 3	+1,-2,+3,-4,-5 4
Rangs signés v	+1,-2,-3,+4,-5 5	+1,-2,-3,-4,+5 6	-1,+2,+3,-4,-5 5	-1,+2,-3,+4,-5 6
Rangs signés v	-1,+2,-3,-4,+5 7	-1,-2,+3,+4,-5 7	-1,-2,+3,-4,+5 8	-1,-2,-3,+4,+5 9
Rangs signés v	+1,+2,+3,-4,-5 6	+1,+2,-3,+4,-5 7	+1,+2,-3,-4,+5 8	+1,-2,+3,+4,-5 8
Rangs signés v	+1,-2,+3,-4,+5 9	+1,-2,-3,+4,+5 10	-1,+2,+3,+4,-5 9	-1,+2,+3,-4,+5 10
Rangs signés v	-1,+2,-3,+4,+5 11	-1,-2,+3,+4,+5 12	+1,+2,+3,+4,-5 10	+1,+2,+3,-4,+5 11
Rangs signés v	+1,+2,-3,+4,+5 12	+1,-2,+3,+4,+5 13	-1,+2,+3,+4,+5 14	+1,+2,+3,+4,+5 15

v	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$P_{H_0}(V_s = v)$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$



18(i) $H_0 : \pi_{.5} = 0$ $\quad H_1 : \pi_{.5} \neq 0$ (unilatéralement)

$$\mathrm{P}_{H_0}(S_4 \geq a) = \sum_{i=a}^{4} \mathrm{P}_{H_0}(S_4 = i) = \mathrm{P}_{H_0}(S_4 \leq 4 - a) = \sum_{j=0}^{a} \mathrm{P}_{H_0}(S_4 = j)$$

$\mathrm{P}_{H_0}(S_4 \geq 4) = \mathrm{P}_{H_0}(S_4 \leq 0) = \frac{1}{16}$

$\mathrm{P}_{H_0}(S_4 \geq 3) = \mathrm{P}_{H_0}(S_4 \leq 1) = \frac{5}{16}$

$H_1 : \pi_{.5} \neq 0$ (bilatéralement)

α_0 : seuil observé

si $S_4 = 0$ ou 4, $\quad \alpha_0 = 2\mathrm{P}_{H_0}(S_4 \geq 4) = \frac{2}{16}$

si $S_4 = 1$ ou 3, $\quad \alpha_0 = 2\mathrm{P}_{H_0}(S_4 \geq 3) = \frac{10}{16}$

Pour les deux H_1, si $S_4 = 2$, on ne rejette pas H_0.

H_0 : il n'y a pas de différence
H_1 : il y a différence (unilatéralement)
$$\mathrm{P}_{H_0}(V_s \geq a) = \sum_{i=a}^{10} \mathrm{P}_{H_0}(V_s = i) = \mathrm{P}_{H_0}(V_s \leq 10 - a) = \sum_{j=0}^{a} \mathrm{P}_{H_0}(V_s = j)$$
$\mathrm{P}_{H_0}(V_s \geq 10) = \mathrm{P}_{H_0}(V_s \leq 0) = \frac{1}{16}$
$\mathrm{P}_{H_0}(V_s \geq 9) = \mathrm{P}_{H_0}(V_s \leq 1) = \frac{2}{16}$
$\mathrm{P}_{H_0}(V_s \geq 8) = \mathrm{P}_{H_0}(V_s \leq 2) = \frac{3}{16}$
$\mathrm{P}_{H_0}(V_s \geq 7) = \mathrm{P}_{H_0}(V_s \leq 3) = \frac{5}{16}$
$\mathrm{P}_{H_0}(V_s \geq 6) = \mathrm{P}_{H_0}(V_s \leq 4) = \frac{7}{16}$

H_1 : il y a différence (bilatéralement)
si $V_s = 0$ ou 10, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 10) = \frac{2}{16}$
si $V_s = 1$ ou 9, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 9) = \frac{4}{16}$
si $V_s = 2$ ou 8, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 8) = \frac{6}{16}$
si $V_s = 3$ ou 7, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 7) = \frac{10}{16}$
si $V_s = 4$ ou 6, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 6) = \frac{14}{16}$
Pour les deux H_1, si $V_s = 5$, on ne rejette pas H_0.

18(ii) $H_0 : \pi_{.5} = 0$ $\quad$ $H_1 : \pi_{.5} \neq 0$ (unilatéralement)
$$\mathrm{P}_{H_0}(S_5 \geq a) = \sum_{i=a}^{5} \mathrm{P}_{H_0}(S_5 = i) = \mathrm{P}_{H_0}(S_5 \leq 5 - a) = \sum_{j=0}^{a} \mathrm{P}_{H_0}(S_5 = j)$$
$\mathrm{P}_{H_0}(S_5 \geq 5) = \mathrm{P}_{H_0}(S_5 \leq 0) = \frac{1}{32}$
$\mathrm{P}_{H_0}(S_5 \geq 4) = \mathrm{P}_{H_0}(S_5 \leq 1) = \frac{6}{32}$
$\mathrm{P}_{H_0}(S_5 \geq 3) = \mathrm{P}_{H_0}(S_5 \leq 2) = \frac{16}{32}$
$H_1 : \pi_{.5} \neq 0$ (bilatéralement)
α_0 : seuil observé
si $S_5 = 0$ ou 5, $\alpha_0 = 2\mathrm{P}_{H_0}(S_5 \geq 5) = \frac{2}{32}$
si $S_5 = 1$ ou 4, $\alpha_0 = 2\mathrm{P}_{H_0}(S_5 \geq 4) = \frac{12}{32}$
si $S_5 = 2$ ou 3, $\alpha_0 = 2\mathrm{P}_{H_0}(S_5 \geq 3) = 1$
H_0 : il n'y a pas de différence
H_1 : il y a différence (unilatéralement)
$$\mathrm{P}_{H_0}(V_s \geq a) = \sum_{i=a}^{15} \mathrm{P}_{H_0}(V_s = i) = \mathrm{P}_{H_0}(V_s \leq 15 - a) = \sum_{j=0}^{a} \mathrm{P}_{H_0}(V_s = j)$$
$\mathrm{P}_{H_0}(V_s \geq 15) = \mathrm{P}_{H_0}(V_s \leq 0) = \frac{1}{32}$
$\mathrm{P}_{H_0}(V_s \geq 14) = \mathrm{P}_{H_0}(V_s \leq 1) = \frac{2}{32}$
$\mathrm{P}_{H_0}(V_s \geq 13) = \mathrm{P}_{H_0}(V_s \leq 2) = \frac{3}{32}$
$\mathrm{P}_{H_0}(V_s \geq 12) = \mathrm{P}_{H_0}(V_s \leq 3) = \frac{5}{32}$
$\mathrm{P}_{H_0}(V_s \geq 11) = \mathrm{P}_{H_0}(V_s \leq 4) = \frac{7}{32}$
$\mathrm{P}_{H_0}(V_s \geq 10) = \mathrm{P}_{H_0}(V_s \leq 5) = \frac{10}{32}$
$\mathrm{P}_{H_0}(V_s \geq 9) = \mathrm{P}_{H_0}(V_s \leq 6) = \frac{13}{32}$
$\mathrm{P}_{H_0}(V_s \geq 8) = \mathrm{P}_{H_0}(V_s \leq 7) = \frac{16}{32}$

H_1 : il y a différence (bilatéralement)
si $V_s = 0$ ou 15, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 15) = \frac{2}{32}$

si $V_s = 1$ ou 14, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 14) = \frac{4}{32}$
si $V_s = 2$ ou 13, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 13) = \frac{6}{32}$
si $V_s = 3$ ou 12, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 12) = \frac{10}{32}$
si $V_s = 4$ ou 11, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 11) = \frac{14}{32}$
si $V_s = 5$ ou 10, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 10) = \frac{20}{32}$
si $V_s = 6$ ou 9, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 9) = \frac{26}{32}$
si $V_s = 7$ ou 8, $\alpha_0 = 2\mathrm{P}_{H_0}(V_s \geq 8) = 1$

33(a)(i)

Les différences sont: 16 87 5 0 -8 90 0 0 31 12
$N_0 = 3$
$S_{10}^* = S_{N-N_0} = S_7 = 6$
$\alpha_0 = 2\mathrm{P}_{H_0}(S_7 \geq 6) = 2(1 - \mathrm{P}_{H_0}(S_7 \leq 5))$
$\alpha_0 = 2\mathrm{P}_{H_0}(S_7 \leq 1) = 2 \times 0.0625 = 0.125 > 0.05$ (table G)
$\Rightarrow$ Au seuil de 5%, on ne rejette pas H_0, donc il n'y a pas de différence entre le ruban et les points de suture après 10 jours.

33(b)(i)

Différences	0	0	0	5	-8	12	16	31	87	90
Rangs	1	2	3	4	5	6	7	8	9	10
Rangs moyens	2	2	2	4	5	6	7	8	9	10
Signes	0	0	0	+	-	+	+	+	+	+

$V_s^* = 44$ $\quad d_0 = 3$
$$\mathrm{E}(V_s^*) = \frac{N(N+1)}{4} - \frac{d_0(d_0+1)}{4} = \frac{10 \times 11}{4} - \frac{3 \times 4}{4} = 24.5$$
$$\mathrm{Var}\,(V_s^*) = \frac{N(N+1)(2N+1)}{24} - \frac{d_0(d_0+1)(2d_0+1)}{24} - 0 = \frac{10 \times 11 \times 21}{24} - \frac{3 \times 4 \times 7}{24} = 92.75$$
$$\alpha_0 = 2\mathrm{P}_{H_0}(V_s^* \geq 44) = 2\mathrm{P}_{H_0}\left(\frac{V_s^* - 24.5}{\sqrt{92.75}} \geq 2.0248\right)$$
$\alpha_0 \simeq 2(1 - \Phi(2.0248)) = 0.04289005 < 0.05$
$\Rightarrow$ Au seuil de 5%, on rejette H_0, donc il y aurait une différence selon ce test.

33(a)(ii)

Les différences sont: -472 102 0 -564 70 -649 -199 -198 -738 -452
$N_0 = 1$
$S_{10}^* = S_{N-N_0} = S_9 = 2$
$\alpha_0 = 2\mathrm{P}_{H_0}(S_9 \leq 2) = 2 \times 0.0898 = 0.1796 > 0.05$ (table G)
$\Rightarrow$ Au seuil de 5%, on ne rejette pas H_0, donc il n'y a pas de différence .

33(b)(ii)

Différences	0	70	102	-198	-199	-452	-472	-564	-649	-738
Rangs	1	2	3	4	5	6	7	8	9	10
Signes	0	+	+	-	-	-	-	-	-	-

$V_s^* = 5$ $\quad d_0 = 1$
$$\mathrm{E}(V_s^*) = \frac{N(N+1)}{4} - \frac{d_0(d_0+1)}{4} = \frac{10 \times 11}{4} - \frac{1 \times 2}{4} = 28$$
$$\mathrm{Var}\,(V_s^*) = \frac{N(N+1)(2N+1)}{24} - \frac{d_0(d_0+1)(2d_0+1)}{24} - 0 = \frac{10 \times 11 \times 21}{24} - \frac{1 \times 2 \times 2}{24} = 95.75$$

$$\alpha_0 = 2\mathrm{P}_{H_0}(V_s^* \leq 5) = 2\mathrm{P}_{H_0}\left(\frac{V_s^* - 28}{\sqrt{95.75}} \leq -2.3505\right) \simeq 2\Phi(-2.3505) = 0.0187487 < 0.05$$

$\Rightarrow$ Au seuil de 5%, on rejette H_0, donc il aurait une différence entre le ruban et les points de suture après 150 jours selon ce test.

6(i)(a)

$\pi = 0.9 \qquad \alpha = 0.01$

$$N \approx \left(\frac{\frac{1}{2}z_\alpha - \sqrt{pq}z_\pi}{p - \frac{1}{2}}\right)^2$$

$z_{0.01} = 2.326348 \qquad z_{0.9} = -1.281552 \qquad p = 0.7476$

$$N \approx \left(\frac{\frac{2.326348}{2} - \sqrt{0.7476(1-0.7476)} \times -1.281552}{0.7476 - \frac{1}{2}}\right)^2 = 48.25 \approx 48$$

6(i)(b)

$\pi = 0.9 \qquad \alpha = 0.02$

$z_{0.02} = 2.053749$

$$N \approx \left(\frac{\frac{2.053749}{2} - \sqrt{0.7476(1-0.7476)} \times -1.281552}{0.7476 - \frac{1}{2}}\right)^2 = 40.90 \approx 41$$

6(ii)(a)

$\pi = 0.95 \qquad \alpha = 0.01$

$z_{0.95} = -1.644854$

$$N \approx \left(\frac{\frac{2.326348}{2} - \sqrt{0.7476(1-0.7476)} \times -1.644854}{0.7476 - \frac{1}{2}}\right)^2 = 57.51 \approx 58$$

6(ii)(b)

$\pi = 0.95 \qquad \alpha = 0.02$

$$N \approx \left(\frac{\frac{2.053749}{2} - \sqrt{0.7476(1-0.7476)} \times -1.644854}{0.7476 - \frac{1}{2}}\right)^2 = 49.46 \approx 49$$

23 Les Z_i sont : 0.14 0.98 0.44 0.37 -0.11 0.15 0.69 -0.05 0.75 1.19 0.01 0.15

$\tilde{\theta} = \underset{1 \leq i \leq 10}{\text{méd}}(Z_i) = 0.26$

Avec un programme analogue à celui utilisé au numéro 2 b), il est possible d'obtenir ces $\left(\frac{Z_i+Z_j}{2}\right)$ ou encore A_i :

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
[1,]	0.140	0.000	0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[2,]	0.560	0.980	0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[3,]	0.290	0.710	0.440	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[4,]	0.255	0.675	0.405	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[5,]	0.015	0.435	0.165	0.13	-0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[6,]	0.145	0.565	0.295	0.26	0.02	0.15	0.00	0.00	0.00	0.00	0.00	0.00
[7,]	0.415	0.835	0.565	0.53	0.29	0.42	0.69	0.00	0.00	0.00	0.00	0.00
[8,]	0.045	0.465	0.195	0.16	-0.08	0.05	0.32	-0.05	0.00	0.00	0.00	0.00
[9,]	0.445	0.865	0.595	0.56	0.32	0.45	0.72	0.35	0.75	0.00	0.00	0.00
[10,]	0.665	1.085	0.815	0.78	0.54	0.67	0.94	0.57	0.97	1.19	0.00	0.00
[11,]	0.075	0.495	0.225	0.19	-0.05	0.08	0.35	-0.02	0.38	0.60	0.01	0.00
[12,]	0.145	0.565	0.295	0.26	0.02	0.15	0.42	0.05	0.45	0.67	0.08	0.15

$$\widehat{\theta} = \underset{1\leq i\leq j\leq 10}{\text{méd}} \left(\frac{Z_i + Z_j}{2}\right) = 0.375$$

$$\overline{\theta} = \frac{1}{12}\sum_{i=1}^{12} Z_i = 0.3925$$

28 Les A_i en ordre croissant :

```
 [1] -0.110 -0.080 -0.050 -0.050 -0.020  0.010  0.015  0.020  0.020
[10]  0.045  0.050  0.050  0.075  0.080  0.080  0.130  0.140  0.145
[19]  0.145  0.150  0.150  0.150  0.160  0.165  0.190  0.195  0.225
[28]  0.255  0.260  0.260  0.290  0.290  0.295  0.295  0.320  0.320
[37]  0.350  0.350  0.370  0.380  0.405  0.415  0.420  0.420  0.435
[46]  0.440  0.445  0.450  0.450  0.465  0.495  0.530  0.540  0.560
[55]  0.560  0.565  0.565  0.565  0.570  0.595  0.600  0.665  0.670
[64]  0.670  0.675  0.690  0.710  0.720  0.750  0.780  0.815  0.835
[73]  0.865  0.940  0.970  0.980  1.085  1.190
```

$A_{(11)}$	$A_{(18)}$	$A_{(23)}$	$A_{(31)}$	$A_{(39)}$	$A_{(40)}$	$A_{(49)}$	$A_{(56)}$	$A_{(61)}$	$A_{(68)}$
0.05	0.145	0.16	0.29	0.37	0.38	0.45	0.565	0.6	0.72

$Z_{(3)}$	$Z_{(4)}$	$Z_{(5)}$	$Z_{(6)}$	$Z_{(7)}$	$Z_{(8)}$	$Z_{(9)}$	$Z_{(10)}$
0.01	0.14	0.15	0.15	0.37	0.44	0.69	0.75

Comparaison de deux moyennes, échantillons appariés : $\mu_D = \mu_Y - \mu_X$

$H_0 : \mu_D = 0$

$$t_0 = \frac{\overline{D}}{s_D/\sqrt{N}} \sim t_{N-1} \qquad \overline{D} = \frac{1}{N}\sum_{j=1}^{N} D_j = \overline{Z} \qquad D_j = Y_j - X_j$$

$\mu_D \in \overline{D} \pm t_{\alpha/2,N-1}\sqrt{s_D/N}$

$s_D = s_Z = 0.1796568 \qquad N = 10$

$\overline{D} - t_{0.01,9}\sqrt{s_D/9} = 0.060$ $\qquad \overline{D} + t_{0.01,9}\sqrt{s_D/9} = 0.725$

$\overline{D} - t_{0.05,9}\sqrt{s_D/9} = 0.173$ $\qquad \overline{D} + t_{0.75,9}\sqrt{s_D/9} = 0.612$

$\overline{D} - t_{0.1,9}\sqrt{s_D/9} = 0.226$ $\qquad \overline{D} + t_{0.1,9}\sqrt{s_D/9} = 0.559$

$\overline{D} - t_{0.25,9}\sqrt{s_D/9} = 0.307$ $\qquad \overline{D} + t_{0.25,9}\sqrt{s_D/9} = 0.478$

$\overline{D} - t_{0.5,9}\sqrt{s_D/9} = 0.3925$ $\qquad \overline{D} + t_{0.5,9}\sqrt{s_D/9} = 0.3925$

Les intervalles de confiance basés le test des rangs signés sont plus “proches” de ceux basés sur le test de Student que ceux basés sur le test des signes.